

# **ON THE HEIGHTS OF WALL-FIRE FLAMES**

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**Introduction.** A correlation between the visible height of a flame and the power output of the flame is useful for a number of reasons. Thus, observation of a fire can permit one to estimate the rate of heat release and therefore the fuel flow rate. Again, an expression for the flame height is needed in order to calculate the upward flame spread rate on walls (Mitler 1990). It is therefore important that such an expression be reliable. The heights of flames from line burners adjacent to walls have been correlated by a number of workers (Hasemi 1986, Grella and Faeth 1975, etc.). The result is generally written in the form

$$z_{fo} = a (\dot{Q}')^{2/3} \quad (1)$$

where  $\dot{Q}'$  is the power output per unit width.  $\dot{Q}'$  is usually given in kW/m, in which case the coefficient  $a$  has been found to lie between 0.052 and 0.06. This result can readily be obtained analytically, assuming that:

- (a) air is entrained into the thermal plume at a rate proportional to the local upward velocity of the gas (the variation of which results from buoyant acceleration of the hot fluid) and that
- (b) the flame ends (i.e., the tip is reached) when enough air has been entrained to permit stoichiometric burning of the fuel (in fact, several times that amount of air is entrained, presumably because of the incomplete mixing of air and fuel in the diffusion flame).

Eq.(1) has also been applied to walls which are pyrolyzing and burning, rather than line burners (Tu and Quintiere 1988), with some success. Eq.(1) must fail at some point, however, since  $z_f$  depends on a power of  $\dot{Q}'$  smaller than unity: suppose, for simplicity, that the wall pyrolyzes at the uniform rate  $\dot{m}''$ . Then if the pyrolyzing section has the height  $z_p$ , the mass-loss rate is

$$\dot{m} = w z_p \dot{m}'' \quad (2)$$

where  $w$  is the width of the pyrolyzing region. This will result in a power output of  $\dot{Q} = \dot{m}H_c$ , where  $H_c$  is the heat of combustion of the volatiles. Thus, the power per unit width is

$$\dot{Q}' = z_p H_c \dot{m}'' \quad (3)$$

and (assuming  $\dot{m}''$  remains approximately constant)

$$z_{fo} \propto (\dot{Q}')^{2/3} \propto z_p^{2/3} \quad (4)$$

(see, for example, Orloff *et al* 1975). Hence as a fire grows upward on a wall, its (pyrolysis) height  $z_p$  grows faster than the flame height  $z_f$ , and eventually, according to Eq.(4), would have to overtake  $z_f$ ; this is unphysical, since a growing flame must always be at least as high as the pyrolysis front. Hence a more detailed analysis of the combustion/fluid dynamic processes must be made.

**Analysis.** In this section, we find a more realistic relationship between the flame and the pyrolysis heights. We do this by constructing a simple model of the rate of combustion as a function of height. We then find an expression for the fuel mass flux as a function of height,  $f(z)$ , in order to relate the flame height with the place where the fuel flux goes to zero.

Refer to Fig. 1a; this is a schematic of a cross-section of the wall flame. The flame is continuous in the region up to  $z_c$ . Above that, the flame begins to be spatially intermittent, and individual flamelets can be observed, which grow less numerous with height. The envelope containing them (labeled PG), however, grows almost linearly in thickness. Moreover, the eye will generally see this entire region as luminous, even though only a fraction of the volume is occupied by flames at any moment. The dashed curve marked "P" contains the thermal plume: it also includes the excess air which is entrained into the plume. An equivalent-thickness flame which is continuous at all heights is shown in Fig. 1b; this is the model flame with which we will work.

The flame height  $z_f$  is generally defined as the height at which the intermittency is 50%. Experiment has shown (Zukoski *et al* 1984) that the intermittency drops linearly from 100% at  $z_c$  to zero at some height above  $z_f$ . For pool fires,  $z_c$  is a weakly increasing function of  $z_f/D$ , where  $D$  is the burner diameter. The mean value is  $z_c = 0.7z_f$  (Cetegen *et al* 1982). For want of wall-fire intermittency data, we assume the same for wall fires. Since the luminosity goes to zero when the intermittency does, both should vanish at  $L_f = 1.3z_f$ . Actually, the intermittency tails off, and truly vanishes at a height  $L_{ff}$  of about  $z_f + 2(L_f - z_f)$ , rather than at  $L_f$ .  $L_{ff}$  is the point where the flux of fuel drops to zero or near zero.

The fuel flow at height  $z$  is  $f(z) = \dot{m}_f(z)$ . We now make the basic assumption that the rate of fuel consumption at any height  $z$  is proportional to the fuel flow there and proportional to the oxygen flow there,  $\dot{m}_{ox}(z)$ .  $\dot{m}_{ox}(z)$ , in turn, is proportional to  $\dot{m}_a(z)$ , the mass flux of that part of the air in the plume which has not yet participated in combustion. Thus, we assume that the reaction rate  $r(z)$  at  $z$  is

$$r(z) = k_o f(z) \dot{m}_a(z) \quad (5)$$

The proportionality coefficient  $k_o$  gives the reaction speed, and depends on both the fluid mixing rate and the chemical reaction rate.  $k_o$  is not given by any simple theoretical argument; it will be determined by appeal to empirical observations, instead (its dimensions are s/m·kg, in SI units).

Even though this is not an exact model for the combustion process, the resulting errors might be expected to over- (or under-) estimate uniformly for all cases. Hence we can still use this formulation to obtain a reasonable guide to the relative behavior of various flames.

We must now find  $\dot{m}_a(z)$ . If fuel came only from a line burner, then  $\dot{m}_a$  would be given by

$$\dot{m}_a(z) = \dot{m}_e(z) - \gamma [f(0) - f(z)] \quad (6)$$

where  $\gamma$  is the stoichiometric ratio of air mass to fuel mass,  $\dot{m}_e(z)$  is the mass flux of air which has been entrained by height  $z$ , and  $f(0)$  is the fuel being emitted by the line burner. Also,  $r(z)$  would be related to the fuel flux *via*

$$r(z) = -f'(z). \quad (7)$$

Now consider the general case, where there is a wall which is burning as well as a line burner source. We confine ourselves to the steady-state case; this is an excellent approximation, since the upward velocity of the hot gases is orders of magnitude greater than that of the usual advancing pyrolysis front. Thus fuel also issues from the (vertical) surface of width  $w$ , between  $z=0$  and  $z=z_p$ . Assume that it does

so at a uniform rate,  $\mu \equiv \dot{m}''(z)$  g/m<sup>2</sup>-s. We define  $f_0 \equiv f(0)$ . Thus if there is a line burner at the origin, of output  $f_0$ , the total fuel flux is

$$\dot{m}_f = f_0 + \mu w z_p \quad (8)$$

The reaction rate and rate of change of fuel flux are now related by

$$f'(z) = \begin{cases} \mu w - r(z) & 0 \leq z \leq z_p \\ -r(z) & z > z_p \end{cases} \quad (9)$$

rather than as in Eq.(7). Note that since  $r(z)$  is continuous at  $z_p$ ,  $f'(z)$  must be discontinuous there.

Finally, the equation for the "clean" air flux must be generalized from Eq.(6) to

$$\dot{m}_a'(z) = \dot{m}_e'(z) - \gamma r(z) \quad (10)$$

We estimate  $\dot{m}_e(z)$  as follows: we assume a turbulent entrainment rate proportional to the local upward velocity, as do Morton *et al* (1956):

$$\dot{m}_e'(z) = \alpha \rho_o w V(z), \quad (11)$$

where  $\alpha$  is the entrainment coefficient,  $\rho_o$  is the density of ambient air, and  $V(z)$  is the upward velocity along the centerline of the fire, at height  $z$ .

As McCaffrey (1979) has shown, the upward velocity along the centerline of a pool fire is

$$V_{pool}(z) = C_1 \sqrt{z}. \quad (12)$$

$C_1$  is given by McCaffrey. We assume  $C_1$  to be the same for wall fires. Lee and Emmons (1961) showed that for a *line* fire, the plume velocity  $V(z)$  reaches an asymptotic value and stays constant thereafter. Thus we simplify the formulation by assuming

$$V(z) \approx \begin{cases} C_1 \sqrt{z} & z \leq z_c \\ C_1 \sqrt{z_c} & z > z_c \end{cases} \quad (13)$$

We also assume, for simplicity, that the transition height  $z_c$  is just where the intermittency begins. From Eqs.(11) and (13),

$$\dot{m}_e' = \alpha \rho_o w C_1 \begin{cases} \sqrt{z} & z < z_c \\ \sqrt{z_c} & z \geq z_c \end{cases} \quad (14)$$

Eq.(14) can be integrated immediately (also see Liburdy and Faeth, 1978). Eqs.(9), (10), and the resulting equation are combined to eliminate  $r(z)$  and to yield Equation (15):

$$f'(z) + k_o w \alpha \rho_o f(z) \int_0^z V(x) dx + k_o \gamma f^2(z) = \begin{cases} \mu w + k_o \gamma f(z)(f_0 + \mu w z) & z \leq z_p \\ k_o \gamma f(z)(f_0 + \mu w z_p) & z > z_p \end{cases}$$

It is expected that when a solution of Eq.(15) is found, we can find the flame-tip height by invoking the condition

$$f(L_f) = 0 \quad (16)$$

Now, it is possible to transform the nonlinear equation (15) into a simple linear first order ODE which admits an analytic solution. This solution shows that, asymptotically,  $f(z)$  falls off exponentially or faster; it does not vanish, however. Instead of using Eq.(16), therefore, we must find what the fractional flow rate  $f(z)/f(0)$  is "predicted" to be, for  $z = L_f$ , with  $L_f$  found some other way; then postulate that that flow rate always corresponds to  $L_f$ . That is, the resulting flow rate is effectively zero.

The analytic solution requires numerical evaluation, so that it is simpler to solve this nonlinear first order ODE numerically in the first place. A program was written to solve Eq.(15) numerically. This requires  $k_o$  and  $\alpha$ , among other inputs. According to Lee and Emmons (1961),  $\alpha = 0.16$  for a line burner. From symmetry, one might guess that against a wall, the entrainment should be halved, so that  $\alpha \approx 0.08$ . Indeed, Grella and Faeth (1975) give  $\alpha = 0.095 \pm .005$

Finally, we must choose  $k_o$ ; the amount of unburned fuel emanating from the tip of a non-sooting, overventilated laminar diffusion flame is only parts per million (Puri *et al* 1991, 1994; Sunderland *et al* 1995). It is reasonable to suppose that this holds *a fortiori* for turbulent diffusion flames. We will find that this condition permits us to choose  $k_o$ .

The program which was written to solve Eq.(15) was run many times, varying the pyrolysis zone height, power per unit width,  $k_o$ , etc. The fuel flow remaining at  $1.6z_f$  (with  $L_f$  defined as  $1.3z_f$ ) was found to be a strong function of  $k_o$ . It was found that for  $k_o = 1.5 \text{ s/m-g}$ ,  $f(1.6z_f)/f(0) = 2.1 \times 10^{-5}$ . Hence  $f(z) = 2.1 \times 10^{-5} \dot{m}_f$  may be taken as the simple criterion for determining the position  $1.6z_f$ . Again, this may well not be the best possible criterion; its inadequacies will be approximately the same for all cases, however, so that we can still use it to obtain a good guide to the relative heights of various flames.

No theoretical guidance was available to determine the dependence of  $z_f$  on  $z_p$ ; rather, it was assumed that a function of the form

$$z_f \sim [z_{fo}^n + z_p^n]^{1/n} \quad (17)$$

which is asymptotically correct in the limits  $z_{fo} \gg z_p$  and  $z_{fo} \approx z_p$ , might fit the data, where  $z_{fo}$  is defined by Eq(1). It was found that  $n = 3$  gives a quite good fit to the numerical "data" from the computer runs. Again, it must be emphasized that (17) can hold no matter what particular correlation is used; that is, whether the height is best given by Eq.(1) or by some other expression.

**Experiments.** There are data on radiant fluxes as a function of height for sintered vertical burners (Markstein and DeRis, 1990), but the flame was cut off by a shield, so that flame-tip heights cannot be found. Tu and Quintiere (1991) burned PMMA and particle-board panels and measured the resulting flame heights. They also summarize data from Hasemi (1986), Saito *et al* (1987), Kulkarni *et al* (1983), and Harkleroad, all of which were consistent with Tu and Quintiere's measurements, within experimental error. As is seen in Fig.2 (adapted from Tu and Quintiere), Eq.(1), with  $a = 0.052$ , gives only a fair fit to their data. Use of Eq.(17) improves the fit in the region  $\dot{Q}' = 10$  to  $30 \text{ kW/m}$ , where the flame height is comparable to the pyrolysis height,  $0.3 \text{ m}$ . The Harkleroad data (not shown) are also better fitted with Eq.(17) than with Eq.(1). More experiments to test this result are desirable, especially with higher walls.

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